

- A company sells peanut butter in cylindrical jars. Marketing research suggests that using wider jars will increase sales. If the diameter of the jars is increased by 25% without altering the volume, by what percent must the height be decreased?
- (A) 10 (B) 25 (C) 36 (D) 50 (E) 60

2004 AMC 10 A, Problem #11—

“Since the volumes are equal, $\pi r_1^2 h_1 = \pi (r_2)^2 h_2$ ”

- **Solution (C)** Let r , h , and V , respectively, be the radius, height, and volume of the jar that is currently being used. The new jar will have a radius of $1.25r$ and volume V . Let H be the height of the new jar. Then

$$\pi r^2 h = V = \pi (1.25r)^2 H, \quad \text{so} \quad \frac{H}{h} = \frac{1}{(1.25)^2} = 0.64.$$

Thus H is 64% of h , so the height must be reduced by $(100 - 64)\% = 36\%$.

OR

Multiplying the diameter by $5/4$ multiplies the area of the base by $(5/4)^2 = 25/16$, so in order to keep the same volume, the height must be multiplied by $16/25$. Thus the height must be decreased by $9/25$, or 36%.

Difficulty: Medium-hard

NCTM Standard: Geometry Standard: Use visualization, spatial reasoning, and geometric modeling to solve problems

Mathworld.com Classification:

Geometry > Solid Geometry > Cylinders > Cylinder