

- A $3 \times 3 \times 3$ cube is formed by gluing together 27 standard cubical dice. (On a standard die, the sum of the numbers on any pair of opposite faces is 7.) The smallest possible sum of all the numbers showing on the surface of the $3 \times 3 \times 3$ cube is
- (A) 60 (B) 72 (C) 84 (D) 90 (E) 96

2002 AMC 10 A, Problem #18— “How many faces are exposed on a given die?”

- **Solution (D)** There are six dice that have a single face on the surface, and these dice can be oriented so that the face with the 1 is showing. They will contribute $6(1) = 6$ to the sum. There are twelve dice that have just two faces on the surface because they are along an edge but not at a vertex of the large cube. These dice can be oriented so that the 1 and 2 are showing, and they will contribute $12(1 + 2) = 36$ to the sum. There are eight dice that have three faces on the surface because they are at the vertices of the large cube, and these dice can be oriented so that the 1, 2, and 3 are showing. They will contribute $8(1 + 2 + 3) = 48$ to the sum. Consequently, the minimum sum of all the numbers showing on the large cube is $6 + 36 + 48 = 90$.

Difficulty: Hard

NCTM Standard: Geometry Standard for Grades 9–12: Use visualization, spatial reasoning, and geometric modeling to solve problems.

Mathworld.com Classification:

Geometry > Solid Geometry > Polyhedra > Cubes