

- There are two values of  $a$  for which the equation  $4x^2 + ax + 8x + 9 = 0$  has only one solution for  $x$ . What is the sum of those values of  $a$ ?

(A)  $-16$       (B)  $-8$       (C)  $0$       (D)  $8$       (E)  $20$

**2005 AMC 10 A, Problem #10— “Try the quadratic equation.”**

- **Solution (A)** The quadratic formula yields

$$x = \frac{-(a+8) \pm \sqrt{(a+8)^2 - 4 \cdot 4 \cdot 9}}{2 \cdot 4}.$$

The equation has only one solution precisely when the value of the discriminant,  $(a+8)^2 - 144$ , is 0. This implies that  $a = -20$  or  $a = 4$ , and the sum is  $-16$ .

OR

The equation has one solution if and only if the polynomial is the square of a binomial with linear term  $\pm\sqrt{4}x^2 = \pm 2x$  and constant term  $\pm\sqrt{9} = \pm 3$ . Because  $(2x \pm 3)^2$  has a linear term  $\pm 12x$ , it follows that  $a+8 = \pm 12$ . Thus  $a$  is either  $-20$  or  $4$ , and the sum of those values is  $-16$ .

**Difficulty:**

**NCTM Standard:** Problem Solving Standard for Grades 9–12: recognize and use connections among mathematical ideas.

**Mathworld.com Classification:**

Algebra > Algebraic Equations > Quadratic Equation