- A set of tiles numbered 1 through 100 is modified repeatedly by the following operation: remove all tiles numbered with a perfect square, and renumber the remaining tiles consecutively starting with 1. How many times must the operation be performed to reduce the number of tiles in the set to one?

(A) 10 **(B)** 11 **(C)** 18 **(D)** 19 **(E)** 20

2002 AMC 10 A, Problem #22— "How many tiles are left after each operation?"

- **Solution (C)** The first application removes ten tiles, leaving 90. The second and third applications each remove nine tiles leaving 81 and 72, respectively. Following this pattern, we consecutively remove $10, 9, 9, 8, 8, \ldots, 2, 2, 1$ tiles before we are left with only one. This requires $1 + 2(8) + 1 = 18$ applications.

OR

Starting with n^2 tiles, the first application leaves $n^2 - n$ tiles. The second application reduces the number to $n^2 - n - (n-1) = (n-1)^2$ tiles. Since two applications reduce the number from n^2 to $(n-1)^2$, it follows that $2(n-1)$ applications reduce the number from n^2 to $(n-(n-1))^2 = 1$, and $2(10-1) = 18$.

Difficulty: Medium-hard

NCTM Standard: Algebra Standard for Grades 9–12: Generalize patterns using explicitly defined and recursively defined functions.

Mathworld.com Classification:
Number Theory > Special Numbers > Sieve-Related Numbers > Sieve