XVI. AMC 8 Practice Questions 06-03

Elisa swims laps in the pool. When she first started, she completed 10 laps in 25 minutes. Now she can finish 12 laps in 24 minutes. By how many minutes has she improved her lap time?

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(A) $\frac{1}{2}$ (B) $\frac{3}{4}$ (C) 1 (D) 2 (E) 3

2006 AMC 8, Problem #3-"Compare lap time in the unit minutes per lap."

Solution (A) When Elisa started, she completed a lap in $\frac{25}{10} = 2.5$ minutes. Now she can complete a lap in $\frac{24}{12} = 2$ minutes. She has improved her lap time by 2.5 - 2 = 0.5 or $\frac{1}{2}$ minute.

Difficulty: Medium NCTM Standard: Algebra Standard for Grades 6-8: use mathematical models to represent and understand quantitative relationships.

Mathworld.com Classification: Number Theory > Arithmetic > Fractions

Initially, a spinner points west. Chenille moves it clockwise $2\frac{1}{4}$ revolutions and then counterclockwise $3\frac{3}{4}$ revolutions. In what direction does the spinner point after the two moves?





(B) east

(C) south

(D) west

(E) northwest

2006 AMC 8, Problem #4— "Ignore the number of complete revolutions."

Solution (B) Ignore the number of complete revolutions because they do not affect direction. One-fourth of the distance around the circle clockwise from west is north. Three-fourths of the distance counterclockwise around the circle from north is east. Chenille's spinner points east.

Difficulty: Medium-easy

NCTM Standard: Geometry Standard for Grades 6–8: analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships. Mathworld.com Classification: Geometry > Transformations > Rotations

The table shows some of the results of a survey by radio station KAMC. What percentage of the males surveyed listen to the station?

	Listen	Don't Listen	Total
Male	?	26	?
Female	58	?	96
Total	136	64	200



	(A) 39	(B) 48	(C) 52	(D) 55	(E) 75
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2006 AMC 8, Problem \#8-"Fill in the missing entries on the table."

Solution (E) Because 200 - 96 = 104 of those surveyed were male, 104 - 26 = 78 of those surveyed are male listeners.

	Listen	Don't Listen	Total
Male	78	26	104
Female	58	38	96
Total	136	64	200

The percentage of males surveyed who listen to KAMC is $\frac{78}{104}\times 100\%=75\%.$

 $\mathbf{Difficulty:} \ \mathrm{Medium}$

NCTM Standard: Problem Solving Standard for Grades 6–8: apply and adapt a variety of appropriate strategies to solve problems.

Mathworld.com Classification: Number Theory > Arithmetic > Fractions > Percent

Jorge's teacher asks him to plot all the ordered pairs (w, l) of positive integers for which w is the width and l is the length of a rectangle with area 12. What should his graph look like?



2006 AMC 8, Problem \#10 "The factors of 12 are the possible lengths of the sides."

Solution (A) When the area of a rectangle is 12 square units and the sides are integers, the factors of 12 are the possible lengths of the sides. In point form, the side lengths could be (1, 12), (2, 6), (3, 4), (4, 3), (6, 2) and (12, 1). Only graph A fits these points.

Difficulty: Medium-hard

NCTM Standard: Number and Operations Standard for Grades 6–8: use factors, multiples, prime factorization, and relatively prime numbers to solve problems. Mathworld.com Classification: Geometry > Plane Geometry > Rectangles

Number Theory > Prime Numbers > Prime Factorization > Factoring

Jeff rotates spinners P, Q and R and adds the resulting numbers. What is the probability that his sum is an odd number?





2006 AMC 8, Problem #17— "The sum of a number from spinner Q and a number from spinner R is always odd."

Solution (B) Because the sum of a number from spinner Q and a number from spinner R is always odd, the sum of the numbers on the three spinners will be odd exactly when the number from spinner P is even. Because 2 is the only even number on spinner P, the probability of getting an odd sum is $\frac{1}{3}$.

Difficulty: Medium-hard NCTM Standard: Data Analysis and Probability Standard for Grades 6–8: understand and apply basic concepts of probability.

Mathworld.com Classification: Number Theory > Integers > Odd Number Probability and Statistics > Probability

How many two-digit numbers have digits whose sum is a perfect square?

	(A) 13	(B) 16	(C) 17	(D) 18	(E) 19
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2006 AMC 8, Problem #11— "The only possible perfect square sums are 1, 4, 9 and 16."

Solution (C) The sum of the digits of a two-digit number is at most 9 + 9 = 18. This means the only possible perfect square sums are 1, 4, 9 and 16. Each square has the following two-digit possibilities:

 $\begin{array}{l} 1:10\\ 4:40,31,22,13\\ 9:90,81,72,63,54,45,36,27,18\\ 16:97,88,79\\ \end{array}$ There are 17 two-digit numbers in all.

Difficulty: Hard

NCTM Standard: Number and Operations Standard for Grades 6–8: Understand numbers, ways of representing numbers, relationships among numbers, and number systems.

 $\label{eq:Mathworld.com} {\bf Mathworld.com} \ {\bf Classification:} \ {\rm Number} \ {\rm Numbers} > {\rm Special} \ {\rm Numbers} > {\rm Square} \ {\rm Numbers}$

The letter T is formed by placing two $2 \operatorname{inch} \times 4 \operatorname{inch}$ rectangles next to each other, as shown. What is the perimeter of the T, in inches?



2006 AMC 8, Problem #6— "Mark each side and add them all."



Solution (C)

The perimeter is 4 + 2 + 1 + 4 + 2 + 4 + 1 + 2 = 20 inches.

OR

Each rectangle has perimeter = 2l + 2w = 2(4) + 2(2) = 8 + 4 = 12 inches. When the two rectangles are positioned to form the T, a two-inch segment of each rectangle is inside the T and is not on the perimeter of the T. So the perimeter of the T is 2(12) - 2(2) = 24 - 4 = 20 inches.

 $\mathbf{Difficulty:} \ \mathrm{Medium}$

NCTM Standard: Geometry Standard for Grades 6–8: analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships. **Mathworld.com Classification:** Geometry > Plane Geometry > Rectangles

Mindy made three purchases for \$1.98, \$5.04 and \$9.89. What was her total, to the nearest dollar?

	(A) \$10	(B) \$15	(C) \$16	(D) \$17	(E) \$18
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2006 AMC 8, Problem #1— "Round each amount to the nearest dollar."

Solution (D) Mindy's total was approximately 2 + 5 + 10 = \$17.

Difficulty: EasyNCTM Standard: Number and Operations Standard for Grades 6–8: compute fluently and make reasonable estimates.Mathworld.com Classification: Number Theory > Rounding

A singles tournament had six players. Each player played every other player only once, with no ties. If Helen won 4 games, Ines won 3 games, Janet won 2 games, Kendra won 2 games and Lara won 2 games, how many games did Monica win?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

2006 AMC 8, Problem #20— **"Each of the six players played 5 games, and each game involved two players."**

Solution (C) Each of the six players played 5 games, and each game involved two players. So there were $\frac{6\cdot 5}{2} = 15$ games. Helen, Ines, Janet, Kendra and Lara won a total of 4 + 3 + 2 + 2 + 2 = 13 games, so Monica won 15 - 13 = 2 games.

 $\mathbf{Difficulty:} \ \mathbf{Medium-hard}$

NCTM Standard: Problem Solving Standard for Grades 6–8: solve problems that arise in mathematics and in other contexts.