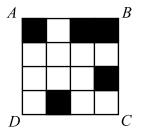
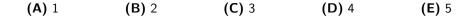
XVI. AMC 8 Practice Questions 05-03

What is the minimum number of small squares that must be colored black so the large square has diagonal BD as a line of symmetry?



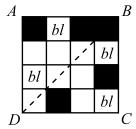


2005 AMC 8, Problem #3---"Look at each of the existing 5 black squares. How many of them need another black square to be symmetric about BD?"

Solution

I X

> (D) For diagonal BD to be a line of symmetry in square ABCD, the four small squares labeled *bl* must be colored black.



Difficulty: Medium

NCTM Standard: Geometry Standard for Grades 6-8: apply transformations and use symmetry to analyze mathematical situations

Mathworld.com Classification: Geometry > Symmetry > Symmetry

A square and a triangle have equal perimeters. The lengths of the three sides of the triangle are 6.1 cm, 8.2 cm and 9.7 cm. What is the area of the square in square centimeters?

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(A) 24 (B) 25 (C) 36 (D) 48 (E) 64

2005 AMC 8, Problem #4— "What is the length of the side of the square?"

Solution

(C) The perimeter of the triangle is 6.1 + 8.2 + 9.7 = 24 cm. The perimeter of the square is also 24 cm. Each side of the square is $24 \div 4 = 6$ cm. The area of the square is $6^2 = 36$ square centimeters.

Difficulty: Medium

NCTM Standard: Geometry Standard for Grades 6–8: analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships Mathworld.com Classification: Calculus and Analysis > Differential Geometry > Differential Geometry of Curves > Perimeter

Soda is sold in packs of 6, 12 and 24 cans. What is the minimum number of packs needed to buy exactly 90 cans of soda?

(A) 4 (B) 5 (C) 6 (D) 8 (E) 15

$\frac{2005 \text{ AMC 8, Problem } \#5---}{\text{``First buy as many 24-packs as possible, then 12, and lastly 6.''}$

Solution

(B) To get the minimum total number, purchase as many 24-packs as possible: three 24-packs contain 72 cans, and 90 - 72 = 18. To get the remaining 18 cans, purchase one 12-pack and one 6-pack. The minimum total number of packs is 5.

 ${\bf Difficulty:} \ {\rm Medium-Easy}$

 ${\bf Mathworld.com\ Classification:\ Number\ Theory > Congruences > Mod}$

 $[\]mathbf{NCTM}$ Standard: Problem Solving Standard for Grades 6–8: solve problems that arise in mathematics and in other contexts

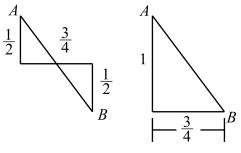
Bill walks $\frac{1}{2}$ mile south, then $\frac{3}{4}$ mile east, and finally $\frac{1}{2}$ mile south. How many miles is he, in a direct line, from his starting point?

(A) 1 (B)
$$1\frac{1}{4}$$
 (C) $1\frac{1}{2}$ (D) $1\frac{3}{4}$ (E) 2

<u>2005 AMC 8, Problem #7</u> "Find the hypotenuse of a triangle with legs $\frac{1}{2} + \frac{1}{2}$ and $\frac{3}{4}$."

Solution

(B) The diagram on the left shows the path of Bill's walk. As the diagram on the right illustrates, he could also have walked from A to B by first walking 1 mile south then $\frac{3}{4}$ mile east.



By the Pythagorean Theorem,

$$(AB)^2 = 1^2 + \left(\frac{3}{4}\right)^2 = 1 + \frac{9}{16} = \frac{25}{16},$$

so $AB = \frac{5}{4} = 1\frac{1}{4}$.

Difficulty: Medium-hard

NCTM Standard: Geometry Standard for Grades 6–8: analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships **Mathworld.com Classification:** Geometry > Plane Geometry > Triangles > Special Triangles > Other Triangles > Right Triangle

Joe had walked half way from home to school when he realized he was late. He ran the rest of the way to school. He ran 3 times as fast as he walked. Joe took 6 minutes to walk half way to school. How many minutes did it take Joe to get from home to school?

(A) 7 (B) 7.3 (C) 7.7 (D) 8 (E) 8.3

2005 AMC 8, Problem #10—

"Covering the same distance three times as fast takes one-third the time."

Solution

(D) Covering the same distance three times as fast takes one-third the time. So Joe ran for 2 minutes. His total time was 6 + 2 = 8 minutes.

Difficulty: Easy

NCTM Standard: Problem Solving Standard for Grades 6–8: solve problems that arise in mathematics and in other contexts Mathworld.com Classification:

Calculus and Analysis > Differential Geometry > Differential Geometry of Curves > Velocity

Big AI, the ape, ate 100 bananas from May 1 through May 5. Each day he ate six more bananas than on the previous day. How many bananas did Big AI eat on May 5?

(A) 20 (B) 22 (C) 30 (D) 32 (E) 34

2005 AMC 8, Problem #12—

"If Big AI had eaten 10 bananas on May 1, how many would he have eaten on May 2?"

Solution

(D) If Big Al had eaten 10 bananas on May 1, then he would have eaten 10 + 16 + 22 + 28 + 34 = 110 bananas. This is 10 bananas too many, so he actually ate 2 fewer bananas each day. Thus, Big Al ate 8 bananas on May 1 and 32 bananas on May 5.

OR

The average number of bananas eaten per day was $\frac{100}{5} = 20$. On May 4, Big Al ate six more bananas than on May 3, and on May 2 he ate six fewer bananas than on May 3. Similarly, on May 5 Big Al ate twelve more bananas than on May 3, and on May 1 he ate twelve fewer bananas. Therefore, the average number of bananas he ate per day, 20, is equal to the number of bananas he ate on May 3. So on May 5 Big Al ate 20 + 12 = 32 bananas.

OR

Let x be the number of bananas that Big Al ate on May 5. The following chart documents his banana intake for the five days.

May 5 May 4 May 3 May 2 May 1 $x \quad x-6 \quad x-12 \quad x-18 \quad x-24$ The total number of bananas Big Al ate was 5x - 60, which must be 100. So Big Al ate $x = \frac{160}{5} = 32$ bananas on May 5.

Difficulty: Medium

NCTM Standard: Algebra Standard for Grades 6–8: understand patterns, relations, and functions Mathworld.com Classification: Number Theory > Sequences > Sequence

The Little Twelve Basketball Conference has two divisions, with six teams in each division. Each team plays each of the other teams in its own division twice and every team in the other division once. How many conference games are scheduled?

(A) 80 (B) 96 (C) 100 (D) 108 (E) 192

$\frac{2005 \text{ AMC 8, Problem } \#14}{\text{"How many total games does each team play?"}}$

Solution

(B) Each team plays 10 games in its own division and 6 games against teams in the other division. So each of the 12 teams plays 16 conference games. Because each game involves two teams, there are $\frac{12 \times 16}{2} = 96$ games scheduled.

Difficulty: Medium-hard

 ${\bf Mathworld.com\ Classification:\ Discrete\ Mathematics > Combinatorics > Permutations > Combination$

NCTM Standard: Problem Solving Standard for Grades 6–8: use solve problems that arise in mathematics and in other contexts

A five-legged Martian has a drawer full of socks, each of which is red, white or blue, and there are at least five socks of each color. The Martian pulls out one sock at a time without looking. How many socks must the Martian remove from the drawer to be certain there will be 5 socks of the same color?

(A) 6 (B) 9 (C) 12 (D) 13 (E) 15

2005 AMC 8, Problem #16—

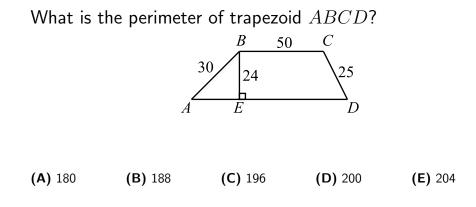
"At most, how many socks of each color can the Martian pull out without having a set?"

Solution

(D) It is possible for the Martian to pull out at most 4 red, 4 white and 4 blue socks without having a matched set. The next sock it pulls out must be red, white or blue, which gives a matched set. So the Martian must select $4 \times 3 + 1 = 13$ socks to be guaranteed a matched set of five socks.

Difficulty: Hard

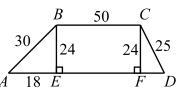
- NCTM Standard: Data Analysis and Probability Standard for Grades 6–8: use understand and apply basic concepts of probability
- Mathworld.com Classification: Probability and Statistics > Probability > Probability



2005 AMC 8, Problem #19— "Use the Pythagorean Theorem to solve for AE."

Solution





By the Pythagorean Theorem, $AE = \sqrt{30^2 - 24^2} = \sqrt{324} = 18$. (Or note that triangle AEB is similar to a 3-4-5 right triangle, so $AE = 3 \times 6 = 18$.)

It follows that CF = 24 and $FD = \sqrt{25^2 - 24^2} = \sqrt{49} = 7$. The perimeter of the trapezoid is 50 + 30 + 18 + 50 + 7 + 25 = 180.

 $\mathbf{Difficulty:} \ \mathrm{Medium-hard}$

NCTM Standard: Geometry Standard for Grades 6–8: analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships **Mathworld.com Classification:** Geometry > Plane Geometry > Quadrilaterals > Trapezoid